Eigenmodes Of Vibration In The Working Process Of A Violin

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There seems to be no correlation between the tuning of the free plate eigenfrequencies and the frequency response curve of the violin. Although free plate modal frequencies change, the assembled instrument modal frequencies remain essentially constant.

When the violin reached its peak in the early 18th century, it was the result of untiring empirical research work. One generation passed its experience to the next—a story of numerous trials and errors.

This paper attempts to show how today's physical understanding and measuring techniques can help the violin maker in his empirical work. It records investigations of how changes in the distribution of mass and stiffness of violin plates affect the acoustical properties of the violin.

Studies by Alonso Moral [1] have investigated the relationship between the free plate mode frequencies and those of the assembled violin by assembling a violin with different pairs of plates. In contrast to that, this paper continuously investigates the modes of *one* violin in the process of producing its plates. The author, a student at the "Staatliche Fachschule für Geigenbau," Mittenwald 1982-86, used as a model for this purpose, the "Cremonese (form 'G')" - a violin by Antonio Stradivari, anno 1715. [2] In fourteen steps of modifying the thickness graduations of the plates, the data on the eigenmodes of the free plates were compared with that of the reassembled instrument at each step.

Construction Process

The aim was to achieve a rather traditional thickness graduation. The fourteen working steps (ST) are given in Figure 1. Modal analysis was done after each of these, and the changes resulting from each of the 14 steps are also described in Figure 1. The grey patterns indicate those areas where the thicknesses of the plates were reduced. The thicknesses (in mm) of those areas after the reduction are given above. The white areas remained unchanged.

In the beginning, the top plate had a constant thickness of 4.0 mm which is far above the usual thickness. In successive steps the thickness was reduced, f-holes cut, bassbar inserted. The final thickness was between 3.2 mm and 1.7 mm.

In the beginning, the back plate had a constant thickness of 5.0 mm and the final thickness was between 4.7 mm and 1.4 mm.

The plates were hollowed out with a precision of +/- 0.1 mm and recorded at 160 points. The spruce for the top plate had 6-7 rings/cm and a density of 372 kg/m^3, the maple for the back plate had 5 rings/cm and a density of 589 kg/m^3.

Measuring Method

The measuring method ("analogous scanning") was described in 1993 at the Stockholm Music Acoustics Conference [3]. The eigenfrequencies are first determined by measuring the input admittance and finding the peaks. Then, to measure a mode shape (see Figure 2), the excitation frequency of the "shaker" is tuned to the selected resonance frequency. The structure is then excited as the very light weight "shaker" (0.1g) is moved over the structure. The response of the structure at the driving point is measured by an accelerometer. Both amplitude and phase can be determined from an oscilloscope which displays Lissajous figures with the excitation signal on one axis and the response signal on the other. A mask provides orientation to the points where data are taken.

This method is less costly and, depending on the application, less time consuming than the classical method of modal analysis, where transfer functions are computer calculated by fast Fourier transforms, curve fitting is done, and mode shapes are demonstrated by means of computer animation. Animations are helpful to get a general idea about the modal behavior of a structure. But as a guide for deliberate modifications of eigenmodes, it is more useful if the mode shapes are described by lines of constant amplitude. Such plots can be obtained from computer based modal analysis by interpolation. Using the method of analogous scanning this information is obtained directly.

Using modal analysis one deals with a single constant digital data package, which is independent of the amount of information that is to be pulled out of it. A typical feature of the method of analogous scanning is that the amount of directly useful information grows with the amount of measurement data put in. So if one already has a general idea about the modal behavior of the violin and intends to modify only a single mode in the construction process a minimum of measuring is necessary.

A disadvantage of this method is that the modes cannot be decoupled. From a certain mode density on when there are many modes in a small frequency range, even with sinusoidal excitation neighboring modes will also be excited. As a consequence, the measured amplitude on the nodal line of a mode being investigated will not be zero. (At higher eigenfrequencies sometimes it is easier to obtain information about the mode shape by watching the change of phase rather than the decrease of amplitude when crossing a nodal line.) For the violin the practical limit is about 1000 Hz. For the violin maker this is not a serious limit for, just as it becomes difficult to recognize separate mode shapes, it also becomes difficult to modify them separately because the nodal lines become closer together and the amplitudes at the antinodes become smaller. What the violin maker can measure by this method corresponds closely to what he can use to tune modes while making the instrument.

There is no frequency limit for measuring the eigenfrequencies, which are determined by their peaks in the admittance curve. Nevertheless, the shifts of eigenfrequencies as given in the following Figures 3 to 5 are only shown in the lower frequency range where it is certain that the peaks are not mixed up. After each working step this requires new measurements of the mode shapes belonging to all peaks.

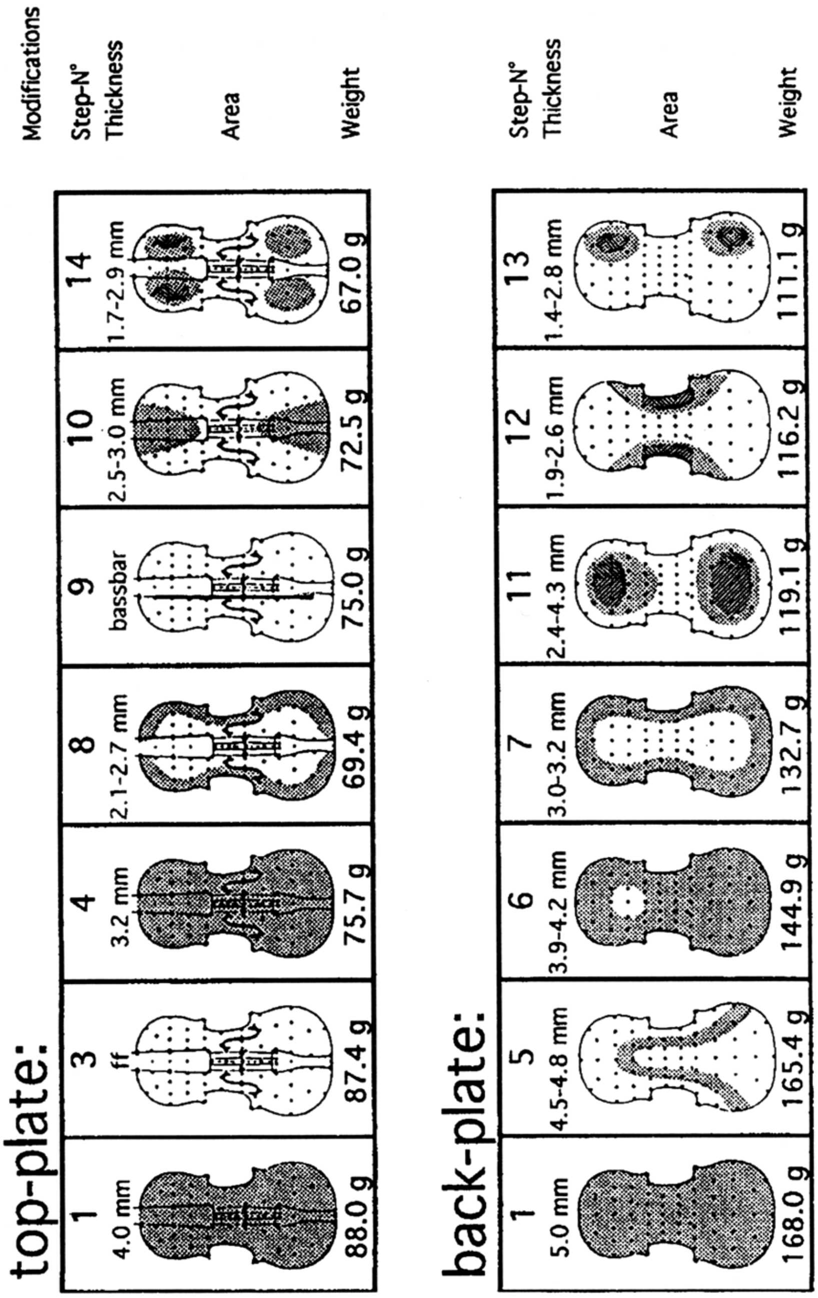

Figure 1 - Workingsteps in the making of a violin

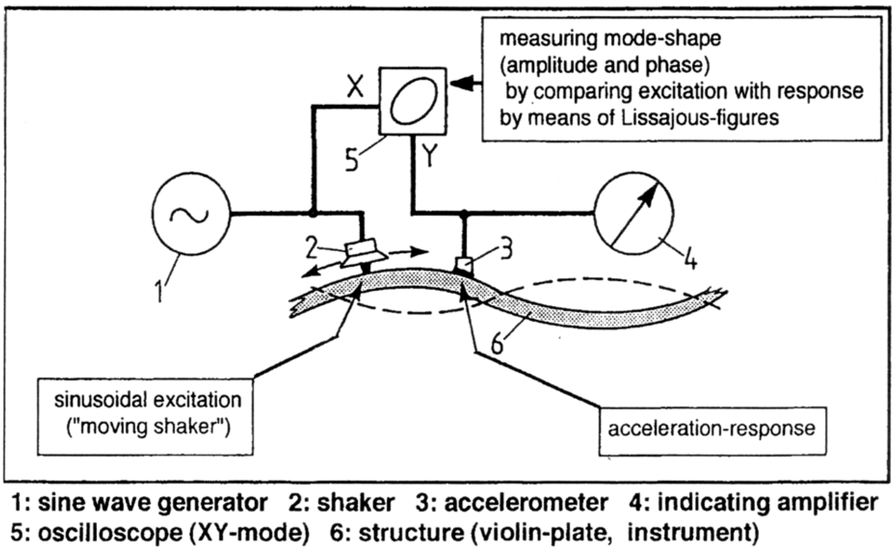

Figure 2 - Measuring Method Using the "Analogous Scanning" Principle

Influences on the measurements

- Comparing the measurement of eigenfrequencies on different days there appeared an average uncertainty of 17 cent, or 1.0% (100 cent is one semitone or 5.95% of frequency).
- The deviation for the first 17 eigenfrequencies (up to 1600 Hz.) when once more gluing the unchanged top plate on the ribs was on average 1.1%.
- Not only the plates had to be glued to the ribs after each working step, but also the soundpost had to be inserted again in order to compare different working steps of the playable instrument. Comparing the eigenfrequencies of two tries of setting the soundpost on the unchanged instrument, the average deviation of eigenfrequencies was 0.6%. This was why as a control, after each plate modification, the instrument was measured both with and without a soundpost.

The eigenfrequencies

Eigenfrequencies (natural frequencies) are those at which a system oscillates when it is excited by a short impulse and is afterwards untouched. If a system is excited by a "shaker," the amplitudes strongly depend on the excitation frequency and the location of the applied force. The highest amplitudes arise if the excitation frequencies correspond precisely with the eigenfrequencies. A full discussion of eigenfrequencies is given by Cremer [4].

The free top plate

The results for the free top plate are shown in Figure 3. The values of the first seven eigenfrequencies are shown at the end of each working step, from which the shifts can be

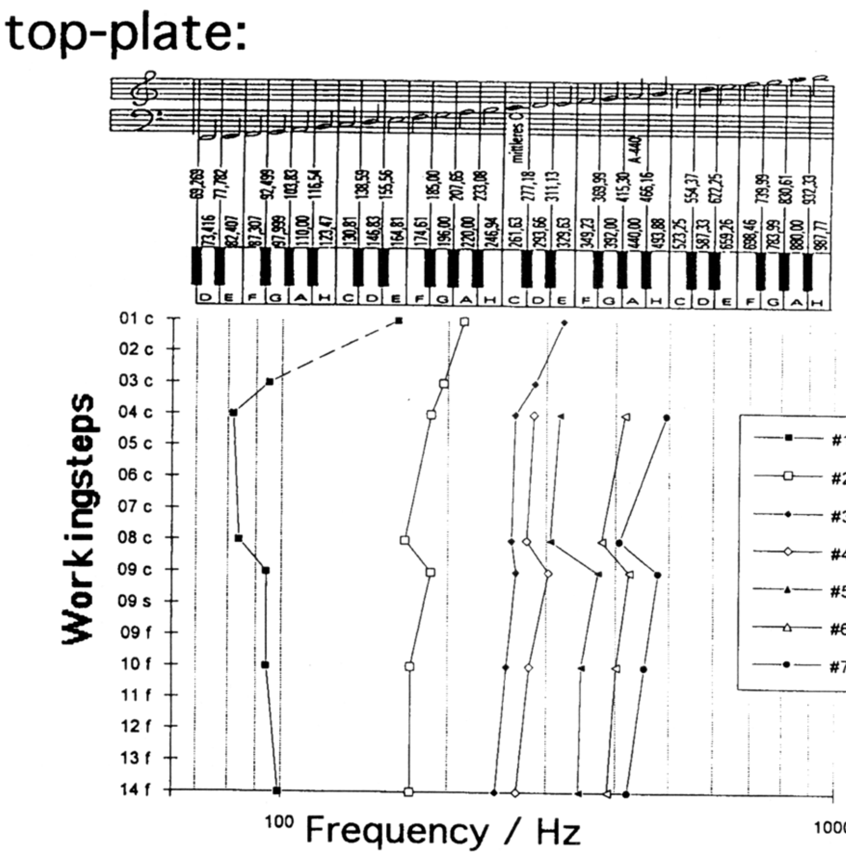

Figure 3.- Shift of eigenfrequencies in the working process of a violin: Top Plate

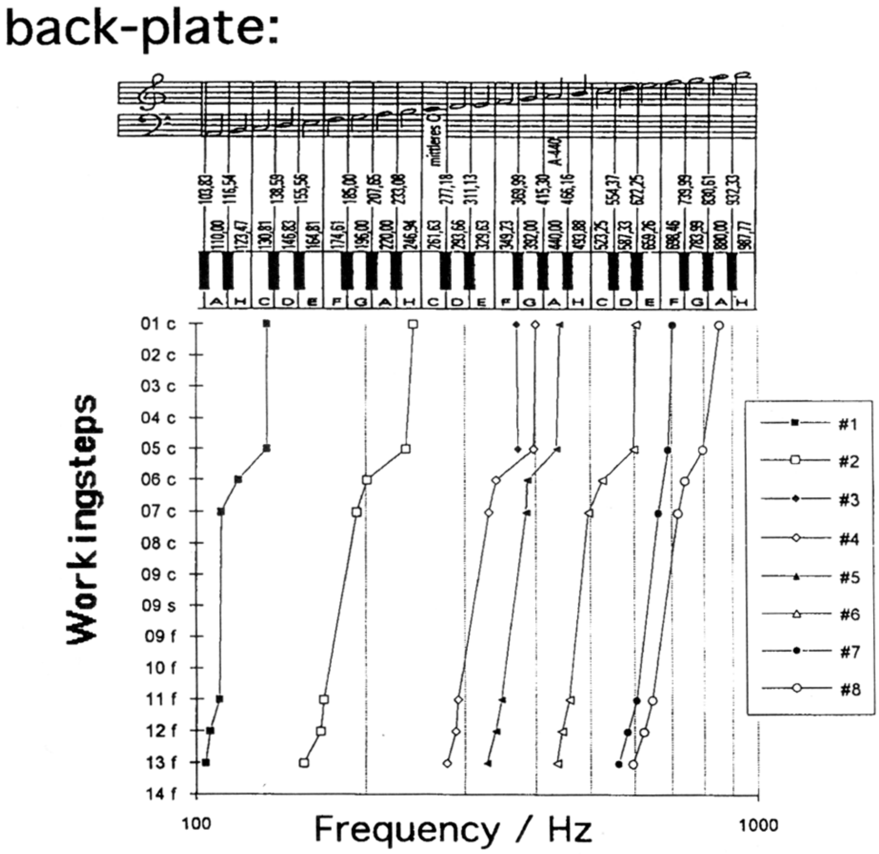

Figure 4 - Shift of eigenfrequencies in the working process of a violin: Back Plate

seen. The logarithmic scale allows one to read the frequency shifts in musical intervals, the "keyboard scale" taken from Pierce [5].

It is immediately obvious that after the completion of the first working step reducing the plate thickness to a uniform 4.0 mm, far above the final thickness, only the first three modes occur. The higher modes that do appear after ST 01, ST 02, and ST 03 are not the same as those after ST 04. Note that the ring mode, called Mode 5, which can be heard as a dominant tap tone, only appears at reduced thicknesses. This corresponds to the results of Jansson [6].

The overall shift of eigenfrequency is between -8.5 semitones (mode #1, from ST 01 to ST 014) and +1.4 semitones (mode #5, from ST 04 to ST 14). The eigenfrequencies of the two modes which are of most interest in plate tuning, #2 and #5, are shifted from 213 to 171 Hz (mode #2), which is from a to f, and from 317 to 344 Hz (mode #5), which is from about e1 to f1. On the whole a decrease of eigenfrequencies and an increase of spectral mode density is caused by the continuing working process on the plate: the frequency ratio of mode #7/#1 in ST 04 is 6.0; in ST 14, only 4.3. After the final step the lowest seven eigenfrequencies cover an interval of about two octaves.

The free back plate

The corresponding diagram for the free back plate is given in Figure 4. As there are no sound holes or bars involved, the effect on the eigenfrequencies is different. Further, in contrast to the top plate, all modes of the back plate can be observed even from the first working step (ST 01).

The overall shift of eigenfrequency is between -3.9 semitones (mode #7, from ST 01 to ST 13) and -7.7 semitones (mode #2, ST 01 to ST 13). The eigenfrequency of mode #2 is shifted from 243 to 156 Hz, which is down from h (b) to e. Mode #5 is shifted from 441 to 328 Hz, which is from a1 to e1. Similar to the top plate, there is a decrease of eigenfrequency values but not an increase of spectral mode density. The frequency ratio of mode #7 to mode #1 in ST 13 is only 5.4. After the final step the lowest seven eigenfrequencies cover an interval of two octaves and a fourth.

The assembled violin

Figure 5 shows the influence of the plate modifications on the lowest twelve eigenfrequencies of the violin as they were assembled fourteen times. The working steps ST 01c to ST 09c describe the corpus with neck and fingerboard, in ST 09s the soundpost was inserted, and ST 09f to ST 14f describe the unvarnished violin with bridge and strings but without chinrest.

After the final step, ST 14f, the lowest seven modes cover a frequency range from 268 to 585 Hz which is only 14 semitones. This range is much smaller than that of its free plates. The peaks are much more closely spaced in the spectrum.

In the beginning (ST 01c) only four modes appear in the observed frequency range, the lowest already having an eigenfrequency of 409 Hz which is an octave into the playing range. Only the f-holes (being cut in ST 02 and 03) make the lower playing range contain any resonances.

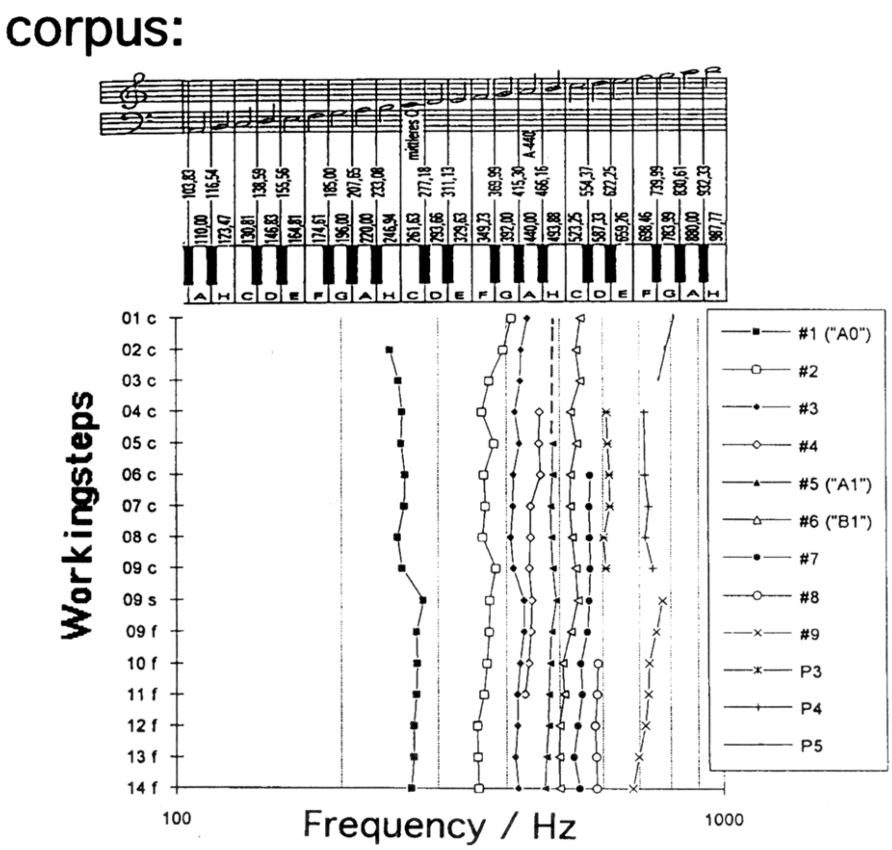

Figure 5 - Shift of eigenfrequencies in the working process of a violin: Corpus

The Helmholtz resonance (also called the f-hole resonance [7]) or "A0 Mode" is created.

With the insertion of the soundpost at ST 12 on, Mode #4 no longer appears as a peak in the spectrum, although it appears until the final working step if the soundpost is not inserted. The reason for this is probably that, due to the frequency shifts caused by inserting the soundpost, its peak is covered by that of Mode #3, which is very close to Mode #4.

Once the f holes have been cut, Mode #5 (the so-called "A1" cavity mode [8]) can be measured along the long portion of the f-hole. The corpus modes Mode #2, Mode #3, and Mode #6 occur during all working steps.

Once the soundpost is inserted, Mode #6 turns out to be the main corpus mode "B1". The "A1 - B1 delta", the interval between the frequencies of these two modes, varies between 0.8 and 2.4 semitones (or 23 to 71 Hz) depending on the working step. For Carleen Hutchins [9] this delta has a crucial meaning as a "controlling factor" for the playing and tonal qualities of the instrument. This interval, depending on the stage of construction, varies from values corresponding (according to her judgment) with those of a typical chamber music instrument after one working step and those of a typical soloist instrument after another working step.

In the range between 350 Hz and 600 Hz there occurs a very dense cluster of modes (#2 to #8). The interval of a major sixth (9 semitones) covers seven different corpus modes. According to Dünnwald [10], this area of resonances is of high tonal importance. But as Figure 5 shows, this mode cluster remains rather constant in frequency. It shows that it is rather independent of the thickness graduations of the plates.

Finally, after all fourteen working steps have been done, the maximum shift of eigenfrequencies is only between 2.4 semitones (Mode #2 from ST 01 to ST 14) and +1.6 semitones (Mode #1 from ST 01 to ST14).

On the discussion of "free plate tuning"

It becomes evident that the eigenfrequencies of the assembled violin have a clearly weaker dependence on the thickness graduations of its plates than the plate eigenfrequencies have under free boundary conditions. An averaging of the magnitudes of all eigenfrequency shifts gives 1.8 semitones per working step for the free top plate, 0.9 semitones for the free back plate, but only 0.3 semitones for the assembled violin itself. This result corresponds well with the finding of Niewczyk and Jansson [11] that resonance frequencies in the assembled instrument are less sensitive to plate thickness than the modes of the free plates.

Figure 6 shows the range of eigenfrequency shifts and the average eigenfrequency shift for each working step. The working step numbers correspond to those given in Figure 1. "sp" represents the influence of the inserted soundpost; "st," the influence of the strings. The lengths of bars show the range of eigenfrequency shifts as measured with respect to the previous working step.

With the exception of the soundpost's influence, the short black bars for the corpus eigenfrequencies (as compared with the grey ones for the free plates) show that the effect on the modes is more unique when the plates are assembled. In other words, the influence of plate modifications on the musical intervals between the peaks in the spectrum is clearly higher for the free plates than for the corpus.

When averaging the eigenfrequency shifts of the free plate modes and of the corpus modes after the same working step (as given by the "average" lines in Figure 6), it is evident that no simple law can be found that would show a correlation between the free plate eigenfrequencies and the corpus eigenfrequencies. The differences in magnitude, range, and average of eigenfrequency shifts call into question the opinion that the free plate eigenfrequencies act as a kind of "barometer" for predicting the nature of the corpus modes.

As a consequence, for violin making it should be emphasized that there seems to be no correlation between a tuning of the free violin plates ("tap tones") and the resulting frequency spectrum of the violin (up to 1000 Hz.)

This makes the meaning of a free plate tuning method questionable. Obviously the thickness graduations of the violin plates do not have the significant influence on the eigenfrequencies of the instrument that is usually assumed.

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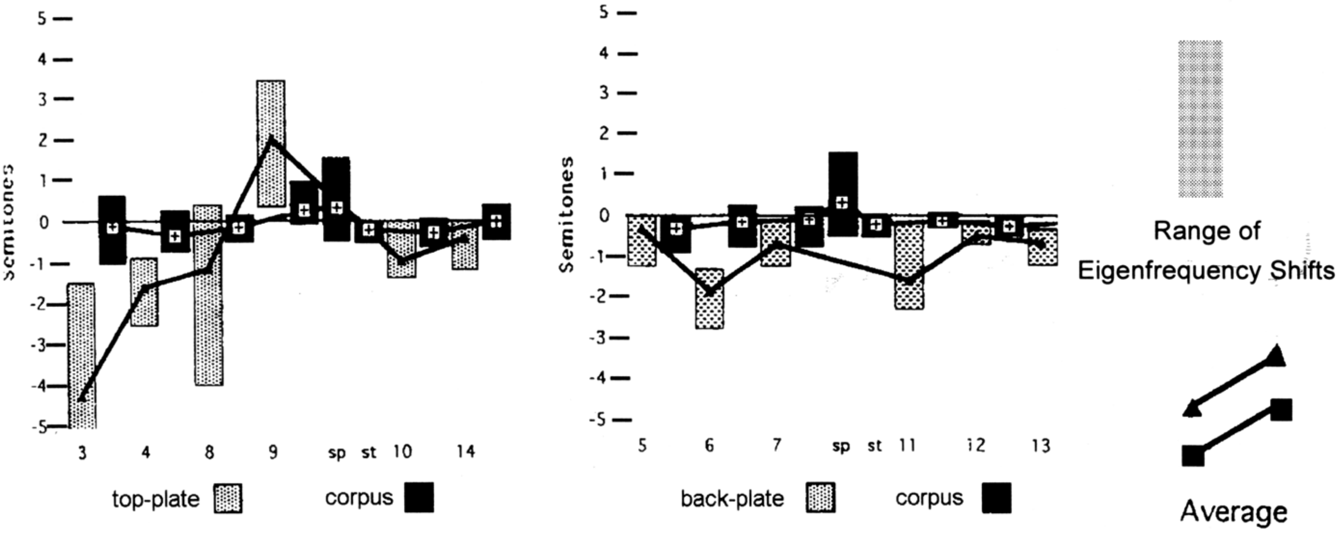

Figure 6 - Shift of eigenfrequencies through the workingsteps in the making of a violin

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