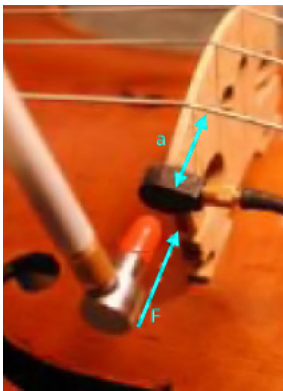


What is a transfer function?

Transfer functions become very important when we want to describe the acoustic properties of a violin since they allow us to actually “see” the resonances of the instrument and thus its unique acoustic profile. We could also refer to the shape of the transfer function as the “resonance profile” of an instrument. Every violin, viola and cello has its own characteristic and unmistakable resonance profile. Just what is a transfer function and how do we go about measuring it?



measuring a transfer function

Fig.: Electronic hammer and acceleration sensor for measuring a transfer function of a violin. The electronic hammer contains a small force measurement cell that is used to measure the fluctuation over time and magnitude of the introduced force F . The acceleration sensor measures the vibration response a . The sensor has a mass of only 0.5g. The transfer function (or frequency response function, FRF) shows us how a violin responds when it is made to vibrate. If the instrument is made to vibrate by a short impulse, it will respond to this stimulus with all of its resonance (intrinsic vibrations). If we wish to determine the FRF, we must measure the impulse excitation (force F) as well as the response (vibration acceleration a). Then, we take the quotient of the response a and the excitation F :

$$\text{FRF} = a(f)/F(f).$$

The short excitation pulse will have a length that is less than a thousandth of a second (the exact timing is measured using a digital signal processor). When viewed in the frequency domain, this short impulse contains all of the frequencies that are needed to excite the instrument.

If the instrument is excited at its resonant frequencies, then a small force F will suffice to produce a large vibration acceleration a . This is due to the fact that at its resonant frequencies the violin is very “mobile” in dynamic terms. We can recognize this large “resonance mobility” by noting the correspondingly large transfer factor a/F . Accordingly, the “resonance profile” has a resonance peak here.

On the other hand, if the instrument is excited at a frequency in the vicinity of which there is no resonance, the excitation force F will produce a low value for the vibration response a . At this frequency, a small transfer factor is

measured accordingly. This can be seen in the form of a notch in the “resonance profile”.

If we plot all of the transfer factors alongside one another vs. increasing excitation frequency (x axis), then the curve we obtain will represent the magnitude of the FRF. We can read off the transfer factor associated with any given frequency from the amplitude axis (y axis).

In other words, the transfer function is a curve that is a function of frequency. We can use it to read off the ratio of the response to the excitation (or the ratio of the acceleration to the force) (a/F) for any given frequency (f):

$$\text{FRF} = a(f)/F(f).$$



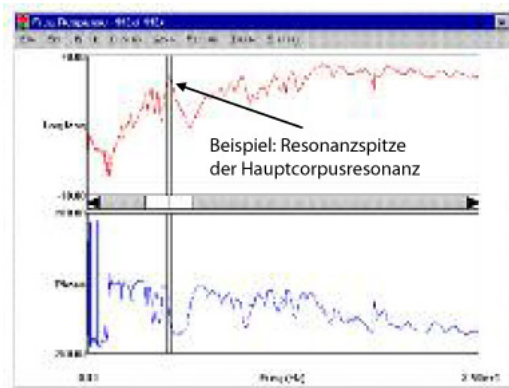
J.B. Fourier

Mathematical computation of the transfer function involves what is known as a Fourier transformation. Here, the time domain representing the two measured signals $a(t)$ and $F(t)$ is transformed into the frequency domain. To make this measurement, we must use what is known as a Fourier analyzer.

J.B. Fourier

J.B. Fourier showed in 1822 as part of his investigation into the nature of heat (“*Theorie analytique de la chaleur*”) that a process having any arbitrary form which repeats over a period equal to T (e.g. the vibration of a bowed violin string) can be constructed entirely from individual harmonic frequency components. The fundamental and harmonic vibrations are known as “Fourier components”.

The Fourier transformation harks back to this principle, and is an algorithm for transforming signals from the time domain into the frequency domain. Using a Fourier transformation, it becomes possible to see all of the frequency components that make up a process that is a function of time.



Resonance profile of a violin

Fig.: Resonance profile of a violin: In the magnitude spectrum (red), each resonance can be seen in the form of a separate “mountaintop”. Here, it is possible to read off the intrinsic resonant frequencies and levels of the resonances. The phase spectrum (blue) provides information about the displacement in time between the excitation and response within an excitation period T .

Through resonance analysis of violins, violas and celli based on measurement of transfer functions, it becomes possible to draw conclusions about the construction, material characteristics and varnish of these instruments.