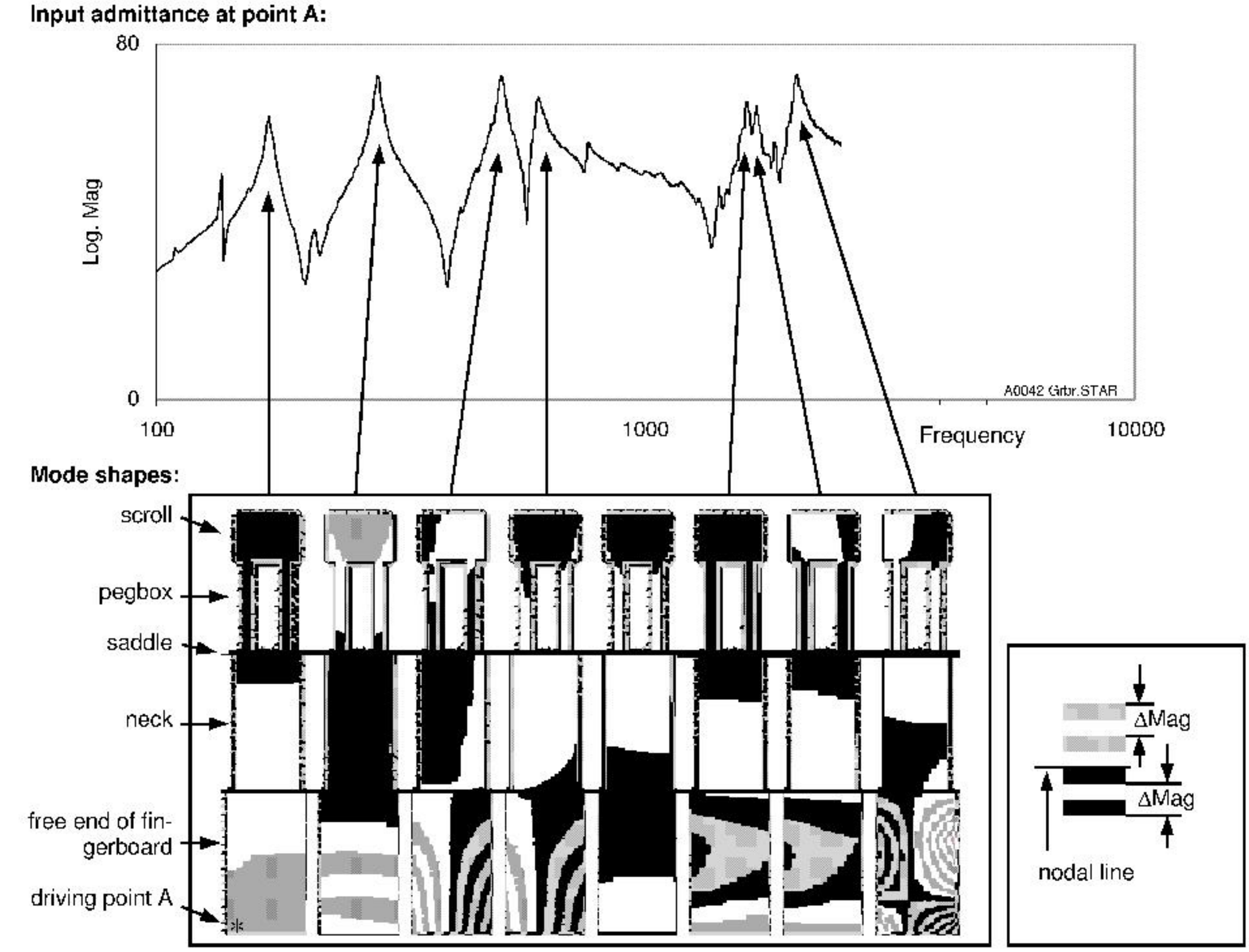
When making new violins and performing tonal adjustments on existing instruments, the results will be outstanding only if and only if all of the subsystems that are capable of vibration (including the tailpiece and fingerboard) are acoustically optimized to the instrument's individual resonance properties. The ability to make adjustments of this sort should be one of the main skills of any good violinmaker. Unfortunately, there has been a lack of systematic research in this area in the past. This article will take a brief look at the research results of the Martin Schleske Master Studio for Violinmaking.

Acoustic analysis of a fingerboard

Acoustic analysis of the resonance profiles of two violins by Antonio Stradivari in our workshop revealed an interesting fact about these two masterpiece instruments: The main corpus resonance ("B1 mode") was doubled in each of them. In the resonance profile of these violins, a split is recognizable. This seems quite extraordinary at first glance. To investigate, we performed comprehensive modal analyses of these two instruments. The mode shapes indicated that the typical eigenmode of vibration of the B1 mode did in fact appear at 513 Hz and just above that at 524 Hz. Upon closer inspection, it turned out that the only system difference between the two modes was in the phase position of the twisting free end of the fingerboard. Clearly, this is related to resonance coupling between the corpus system and the fingerboard subsystem.

Modal analysis of the fingerboard when glued to the instrument (on another instrument in this case) revealed a number of subsystem modes which are largely independent of the instrument corpus.

Fig. 1 shows these fingerboard modes. The frequency response can be determined from the curve of the input admittance (at driving point A). Modal analysis allows us to see the eigenmodes of vibration. They are displayed under the frequency response using contour lines. The fingerboard subsystem has eight modes in the frequency range up to 2500 Hz. Mode #2 is a bending mode of vibration of the free end of the fingerboard (="B0"); modes #3 and #4 are torsion modes of the free end of the fingerboard; modes #6 and #7 are much higher frequency bending vibrations with a nodal line within the free end; and finally mode #8 is a torsion mode with a nodal line crossing within the free area of the fingerboard.



Mode:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Frequency (Hz)	**170 Hz**	**283 Hz**	**507 Hz**	**603 Hz**	**759 Hz**	**1620 Hz**	**1680 Hz**	**2030 Hz**
Damping	1.0 %	1.0 %	1.3 %	1.6 %	1.0 %	1.1 %	1.1 %	1.5 %
max. Mag. (v/F)	1.9E+01	1.7E+01	1.4E+02	1.2E+02	8.1E+00	4.6E+01	3.9E+01	1.9E+02
min. Mag. (v/F)	-3.6E+01	- 9.5E+01	- 1.1E+02	- 7.6E+01	- 1.4E+01	-4.0E+01	-3.8E+01	-2.2E+02

The shown mode-shapes result from the residues of the Frequency Response Functions (FRF); with FRF=v(f)/F(f) (admittance).
Amplitude difference between adjusting amplitude rings ΔMag=25mHz/Ns.

Fig. 1: Frequency response of the input admittance of the glued-on violin fingerboard (above) and related shapes of the eigenmodes of vibration (below). Experimental modal analysis measurement in the Martin Schleske Master Studio for Violinmaking.

Note: The driving point of the measurement is localized at the lower right corner of the freely vibrating end of the fingerboard.

What we find is that through proper thickness graduation of the fingerboard, it becomes possible to couple one of the torsion modes TF of the fingerboard (#3 or #4) with a corpus mode. When the instrument is in its playing state, the result is a spectral split of the corpus mode which is visible in the resonance profile in the form of a desired widening of the corpus resonance region in the fundamental range of the a-string. Much more common is what is known as “A0-B0 coupling". In this case (and with the two Stradivarius instruments mentioned above), mode #2 shown in Fig. 16 (bending mode of vibration, here at 283 Hz) is coupled with the Helmholtz resonance A0. Woodhouse [1] has provided a very detailed explanation of the A0-B0 coupling phenomenon.

Based on our experience in the workshop, we have noted that instruments with resonance coupling (A0-B0 coupling and if possible also TF coupling) are very popular among musicians due to their enhanced playability (= “lively”, “resonant”). Note the similar opinions expressed by Hutchins [2] and Woodhouse [1].

The frequency response of the structure-born sound plotted in Fig. 2 represents an example of successful thickness graduation of the fingerboard which leads to TF coupling as well as A0-B0 coupling.

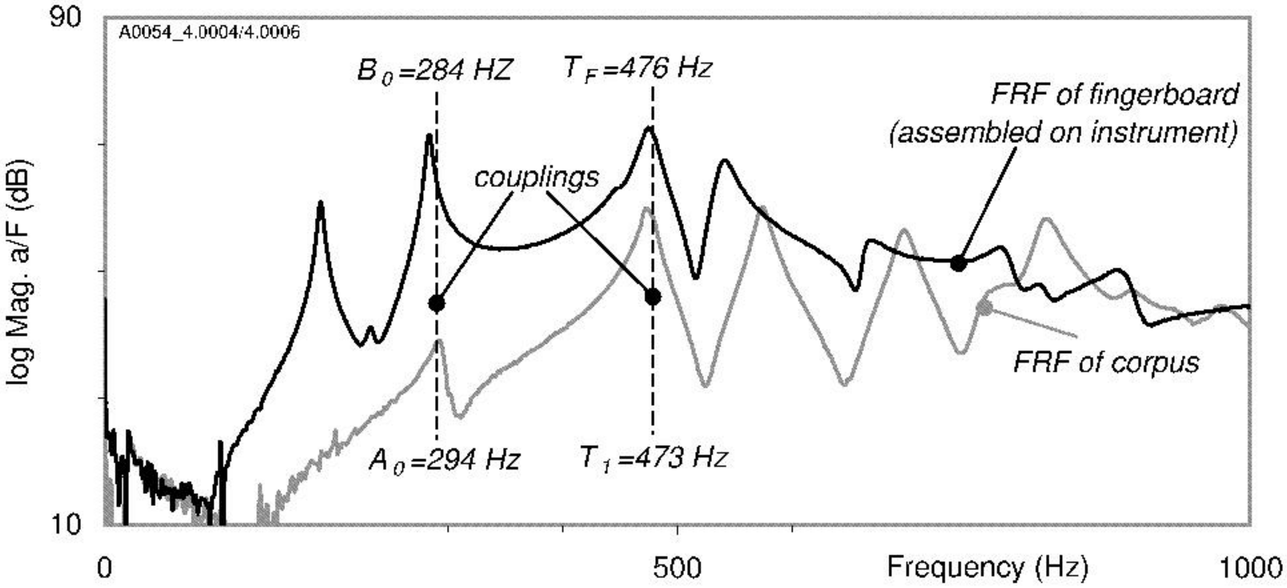


Fig. 2: Frequency response of the corpus resonance of a violin (gray) and the frequency response of the glued-on fingerboard modes (black). As evidenced by the vertical dashed lines, it was possible to achieve double resonance coupling here. ©M. Schleske

The black frequency response is a transfer function between the two free ends of the glued-on fingerboard. The f-holes were covered with foamed material to avoid coupling with the A0 mode so as to be able to determine the actual eigenfrequency of mode #2 (“B0”) of the fingerboard.
The gray curve represents an input accelerance (a/F where a = acceleration and F = excitation force) of the corpus (measured at the location of the left bridge foot). The free end of the fingerboard was restrained in order to avoid coupling with B0 and TF so as to be able to determine the actual eigenfrequencies of the A0 mode and the corpus resonances.

The final result reveals an eigenfrequency match (vertical lines) between A0 and B0 as well as between TF (fingerboard torsion) and the T1 corpus mode. This means the fingerboard subsystem is optimally set for double coupling with the corpus. This will give the instrument a perceptible gain in terms of its “liveliness” and “resonance”.

We have found the following empirical formulae useful in the workshop for simplifying the process of thickness graduation and individual adjustment of the fingerboard:

The fingerboard’s eigenfrequency and the violin’s B0 mode

The lowest eigenmode of vibration of the free fingerboard (not yet glued on) is very similar in terms of its mode shape to the low-frequency B0 eigenmode of vibration of the (not yet playable) violin: Both modes are characterized by a high-amplitude longitudinal bending vibration of the fingerboard. How do these two eigenmodes of vibration look?

1. The lowest eigenmode of vibration of the free fingerboard is a longitudinal bending vibration (fF#1n=2) with two nodal lines in the transverse direction. (There is an easy way to hear this eigenmode of vibration: Hold the fingerboard between your thumb and index finger about one fourth of the way from the end and tap on the end. The lowest tap tone which is heard is the eigenfrequency of the longitudinal bending vibration).

2. The eigenmode of vibration of the B0 mode is represented in Fig. 1 above as mode #2.

Since the mode shapes of the two modes are very similar, any modification to the fingerboard will influence the eigenmodes of vibration of both modes in a similar manner. Examples of modifications of this sort are:

- A change in the thickness distribution
- A change in the lower concavity
- A change in the fingerboard length
- A change in the material quality

By making modifications of this sort, an experienced violinmaker can adjust the stiffness-to-mass distribution of the fingerboard and thus the eigenfrequency of the future B0 mode. A new or modified fingerboard will be successful in acoustic terms if in the completed instrument the B0 mode lies at the frequency of the A0 mode (Helmholtz resonance). In other words, the violinmaker can acoustically match the fingerboard to the instrument so that the instrument will end up having the desired resonance coupling. The result will be an instrument with a noticeable boost in “liveliness” and “resonance”. This means an experienced violinmaker will want to repeatedly check the lowest tap tone while working on the free fingerboard. On the other hand, it takes experience to understand the frequency relationship between the lowest eigenmode of vibration of the free fingerboard and the future B0 mode. The reason is that the B0 mode (which is the objective of the matching process) is not apparent until after the fingerboard is glued on. Due to the similarities between the mode shapes (described above), the relationship between the two eigenfrequencies is nearly linear. Since it is easily heard, the lowest eigenfrequency of the free fingerboard can be a useful aid for estimating the B0 eigenfrequency of the future (completed) instrument when making the fingerboard and graduating its thickness.

Now, however, what is the frequency relationship between the lowest tap tone of the free fingerboard and the B0 eigenfrequency? Since the realization of the neck and the mass of the scroll and pegs also influence the B0 eigenfrequency to a certain degree, the frequency relationship we cited must include a certain error tolerance for different instruments. With the instruments we make (which always have relatively similar scroll and neck thickness graduations), the eigenfrequency of the lowest bending mode of vibration of the free fingerboard (#1n=2) lies above the eigenfrequency of the B0 mode of the completed instrument by a factor of 1.67 to 1.68. This leads to our first important empirical formula:

$$fF\#1_{n=2} = 1.67\, fB_0$$

Where:
fF#1n=2: Eigenfrequency of the first bending mode of vibration of the freely supported fingerboard (2 nodes)
fB0: Eigenfrequency of the B0 mode of the completed violin (longitudinal bending vibration)

Since this frequency relationship is roughly equal to a major sixth, the free fingerboard should be tuned by about this interval above the A0 mode in order to achieve good A0-B0 coupling after the fingerboard is glued on. Determining the A0 eigenfrequency (e.g. by blowing across the f-holes) has to be done with the soundpost installed since it shifts the A0 eigenfrequency upwards by a considerable amount.
Note: Depending on exactly how the neck and scroll are built, this frequency relationship can vary to some extent since the B0 mode is also dependent on the stiffness and mass of the neck and the scroll.

The length of the fingerboard

It is possible to coordinate B0 and TF to a certain degree by adjusting the fingerboard length. The following formulae were obtained by curve fitting the data points that were obtained through successive shortening of the glued-on fingerboard and measurement of the resulting B0 and TF eigenfrequencies.

Increasing the B0 eigenfrequency by shortening the fingerboard:

$$\Delta fB_0 = \Delta L(0.1531\Delta L + 1.3097)$$

Where:
Δf B0: Increase in the eigenfrequency of the B0 mode as a percentage
ΔL: Decrease in the fingerboard length as a percentage
The determination coefficient of this empirical curve fit is R=0.9979.

Example: Shortening the fingerboard length from 270 mm to 265 mm (a 1.85% reduction). Using formula II, we obtain:
The increase in the B0 eigenfrequency (in %) is equal to 1.85*(0.1531*1.85+1.3097)=2.95%.
In other words, if the B0 mode has a frequency of 250 Hz, shortening the fingerboard length by 5 mm will increase it to 257.4 Hz.

Increasing the TF eigenfrequency by shortening the fingerboard:

$$\Delta f T_F = 1.6246 * \Delta L$$

Where:
Δf TF: Increase in the eigenfrequency of the TF mode as a percentage
ΔL: Decrease in the fingerboard length as a percentage
This empirical curve fit also provided a high determination coefficient (R=0.9972).
Example: Decrease in the fingerboard length by 5 mm (or by ΔL=1.85%). Using formula III, we obtain:
The increase in the TF eigenfrequency (in %) is equal to 1.6246*1.85%=3%.
In other words, if the TF mode has a frequency of 510 Hz, shortening the fingerboard length by 5 mm will increase it to 525.3 Hz.

This formulae are a very useful estimation tool for relatively simple frequency adaptations of the fingerboard. However, the most "frequency-sensitive" work step when making a fingerboard is the concave hollowing out of the bottom side (length and depth). The diagram shown in Fig. 2 represents a successful adjustment of a violin fingerboard in the Martin Schleske Master Studio for Violinmaking. The resonance coupling that was achieved resulted in an instrument that is very "lively" and "resonant" when played.
Of course, there are many other elements of the violin that could be subjected to parameter studies of the type described here for the fingerboard. Among the various "simulation experiments" which have examined the effects of parameter changes in the bass bar, the thickness graduation, etc. using the finite element method, we would like to mention in particular the work by Rodgers [3].

Further reading:

[1] Woodhouse, J.: The Acoustics of "A0-B0 Mode Matching" in the Violin, J. Acoust. Soc. Amer. 84 (1998) 947-956.

[2] Hutchins, C.M.: Effect of an air-body coupling on the tone and playing qualities of violins, J. Catgut Acoust. Soc. Nov 1985, pp. 12-15.

[3] Rodgers, O.E.: On the function of the violin bass bar, J. Catgut Acoust. Soc. Nov 1999, pp. 15-19.

[4] Rodgers, O.E.; Anderson P.: Finite Element Analysis of a Violin Corpus, J. Catgut Acoust. Soc. Nov 2001, pp 13-26.